A GENERAL NUMERICAL ANALYSIS OF THE SUPERCONDUCTING QUASIPARTICLE MIXER

R.G. Hicks†, M.J. Feldman, and A.R. Kerr
NASA/Goddard Institute for Space Studies
2880 Broadway, New York, NY 10025

Abstract

Virtually all analyses of the superconductor-insulator-superconductor (SIS) quasiparticle mixer have employed the quantum theory of mixing in its three-frequency approximate form. This approximation is valid only in the limit of very large junction capacitance. For finite capacitance, these analyses may be seriously in error. To remedy this, a computer program has been developed which uses the quantum theory of mixing in its most general form, treating the large-signal properties of the mixer in the time domain. The terminating impedances at the harmonics of the local oscillator frequency and their sidebands can be arbitrary. Using this analysis, the effect of finite junction capacitance on one SIS mixer's performance is described. This gives an insight into the range of validity of the three-frequency model.

Introduction

The superconductor-insulator-superconductor (SIS) quasiparticle mixer is now competitive with conventional Schottky mixers for very low noise millimeter-wave receivers. SIS receivers are currently in routine use at at least four radio astronomy observatories.

The rapid improvement in SIS mixer performance has been made possible by the application of the quantum theory of mixing, developed by J.R. Tucker.[1,2] In almost every case, this theory has been used in the three-frequency approximation, the sole exception being the five-frequency analysis described in Ref. 3. In the three-frequency approximation, the crucial assumption is that the local oscillator (LO) voltage is perfectly sinusoidal, or equivalently, that every harmonic of the LO frequency, ω_p, is short-circuited. With this assumption, the general equations of the quantum theory of mixing[1] can be cast into a relatively simple form for either analytical or numerical analysis of an experimental mixer.

The sinusoidal LO assumption would appear to be justified for those cases in which the impedance of the mixer circuit is dominated by the geometrical capacitance C_J of the SIS junction; many SIS mixers have employed junctions which have a susceptance $\omega_p C_J$ which is from 2 to 10 times as large as their normal-state conductance $1/R_N$. This is indeed the case for one mixer using a 14-junction array with $\omega_p R_N C_J \sim 10$, which showed quite precise agreement with the quantum theory predictions in the three-frequency approximation.[4] Nevertheless, there is considerable evidence, presented in Ref. 5, that the three-frequency approximation is not always valid for junctions with an $\omega_p R_N C_J$ product below perhaps 4, in the sense that harmonic conversion effects appear seriously to degrade the mixer's performance. For the case of mixers with $\omega_p R_N C_J < 1$ the three-frequency approximation should certainly not be valid, although, for lack of an alternative, it has been used to analyze these mixers as well.[6,7] Note, however, that Phillips and Dolan,[8] whose junction capacitance, though not stated, is presumably small, found their experiments in good agreement with the quantum theory in the three-frequency approximation. A theoretical review of published experimental data on SIS mixers is reported in Ref. 9.

A general method of numerical analysis for SIS mixers, which allows arbitrary terminating impedances for all the harmonic frequencies, is presented in this paper. This analysis makes it possible to examine the range of validity of the three-frequency results of the quantum mixer theory. It also serves as a general tool for exploring the behavior of SIS mixers with small capacitances for which the quasiparticle nonlinearity should render the junction voltage distinctly nonsinusoidal.

This project is of immediate practical importance. A large parasitic capacitance will certainly have deleterious effects on the performance of any mixer, unless it can be tuned out at the signal frequency. Such tuning is difficult for a large capacitance (i.e. $\omega_p R_N C_J \sim 10$) at millimeter wavelengths. Therefore, it is desirable to design an SIS mixer with as small a capacitance as possible, consistent with superior mixer performance.

Discussion of Methods

The solution of the general equations of the quantum mixer theory in the sinusoidal LO approximation is straightforward,[1,2] and is routinely done in the frequency domain. When the LO harmonics are not shorted, however, it becomes necessary to use an iterative procedure until a self-consistent solution is obtained for the periodic large-signal LO voltage waveform across the junction. Once the LO waveform has been determined, computation of the small signal properties of the mixer at the signal, image, IF, and harmonic sideband frequencies is a relatively simple generalization of the three-frequency solution.

For the general solution to the large signal problem, it is convenient to formulate the equations of the quantum mixer theory in the time domain. This has been done for an SIS junction by Tucker, Ref. 10, Appendix C. The inputs to this set of equations are the measured dc I-V characteristic of the junction $I_{dc}(V)$ and the voltage waveform impressed across the junction, $V(t)$. The output from these nonlinear equations is the current waveform, $I^{NL}(t)$.

The first and most difficult step in analyzing an SIS mixer is the determination of the periodic LO voltage waveform across the junction. The solution must be consistent with the arbitrary terminating impedances seen by the junction at all the LO harmonics. To solve this problem, this paper will rely upon the methods developed for the general analysis of classical Schottky-barrier diode mixers.

Two techniques have been found useful in analysing a Schottky diode mixer in which the LO waveform at the diode contains many harmonics. Kerr[11] developed the multiple reflection technique, in which a hypothetical transmission line of arbitrary characteristic impedance, Z_0, is introduced between the nonlinear element and the linear embedding circuit. This method has been found to converge in all cases tested; the rate of convergence depends on the proximity of the embedding impedances at each LO harmonic to Z_0.[12] There is no dependence upon the estimate of the initial conditions. The other technique, developed by Hicks and Khan,[12,13] consists of two dual methods, the voltage update and the current update methods. This has also converged in all cases

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tested, and the rate of convergence depends upon the proximity of the embedding impedances at each LO harmonic to either short circuits or open circuits.

The essential elements of both the voltage update method and the multiple reflection method have been adapted to the general analysis of SIS mixers, as presented in this paper. The voltage update method is preferred, especially for junctions with $\omega_p R_N C_J > 0.5$. This is because the junction capacitance, which is treated as part of the SIS external circuitry, provides an embedding impedance, which for higher harmonics, approaches a short circuit. Moreover, the use of the voltage update method permits the nonlinear SIS equations to be solved in a simpler form. This is the voltage-input, current-output mode which is obtained directly from the equations of Ref. 10. The multiple reflection technique and the current update method require that the SIS nonlinear equations be solved on a current-input, voltage-output basis, which adds an extra iterative loop to the process. Thus, the voltage update method has been used here since it usually results in an order of magnitude decrease in CPU time over that required by the multiple reflection technique. (However, in cases for which $\omega_p R_N C_J$ is less than about 0.5, the latter method is often easier to use and for this reason is included in the program described in Ref. 14).

Analysis

The equivalent circuit used in this analysis is shown in Fig. 1. The following input data are required: (1) the embedding impedances at the LO fundamental, its harmonics and the sideband frequencies, (2) the SIS tunnel junction capacitance and measured dc I-V curve, and (3) the magnitudes of the dc and LO sources providing power to the circuit. Using the iterative algorithm described below, the pumped voltage and current waveforms at the junction are determined. A linear small-signal conversion matrix is then derived from the pumped waveform. This gives the conversion efficiency, the signal input impedance and the IF output impedance. Finally, the shot and thermal noise sources are introduced and the same linear small signal analysis is used to determine the mixer noise temperature.

Large Signal Analysis

For the large signal analysis using the voltage update method, the circuit of Fig. 1 is bisected at the linear-nonlinear interface and each half is treated separately. The nonlinear portion of the circuit is treated in the time domain while the embedding network, which is taken to include the geometrical capacitance of the junction, is treated in the frequency domain.

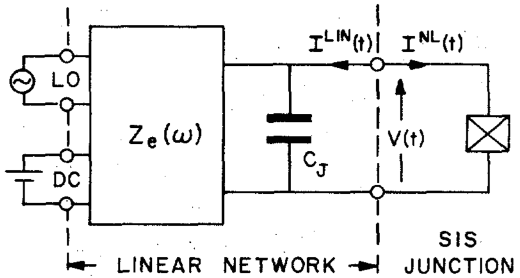

Figure 1. SIS Mixer Circuit

The time domain equations for the nonlinear quasiparticle current through an SIS junction are given by Tucker[10]:

$$I^{NL}(t) = \frac{V(t)}{R_N} + \mathrm{Im}\left\{U^*(t) \int_{-\infty}^{t} \chi(t-t')U(t')dt'\right\} \quad (1)$$

$$\chi(t) = \frac{2}{\pi}\int_0^\infty \left[I_{dc}\left(\frac{\hbar\omega}{e}\right) - \frac{\hbar\omega}{eR_N}\right] \sin \omega t \, d\omega \quad (2)$$

$$U(t) = \exp\{i\phi(t)\} \quad (3)$$

$$\phi(t) = -\frac{e}{\hbar}\int_{-\infty}^{t} V(t')dt' \quad (4)$$

where I_{dc} (V) is the measured dc I-V characteristic of the junction; R_N is the normal resistance of the junction; V(t) is the instantaneous voltage across the junction; and $I^{NL}(t)$ is the instantaneous quasiparticle tunneling current.

These time domain equations have recently been used to investigate the possibility of chaotic phenomena in SIS mixers.[15]

The frequency domain equations for the linear embedding network are found from (referring to Fig. 1):

$$\frac{V_n}{I_n^{LIN}} = \left[Z_e(n\omega_p) \ // \ Z_c(n\omega_p)\right] \quad n = 2,3,\ldots,N. \quad (5)$$

$$\frac{V_1 - V_{LO}}{I_1^{LIN}} = \left[Z_e(\omega_p) \ // \ Z_c(\omega_p)\right] \quad (6)$$

$$\frac{V_0 - V_{dc}}{I_0^{LIN}} = \left[Z_e(0)\right], \quad (7)$$

where V_n and I_n^{LIN} are the amplitudes of the Fourier components of V(t) and $I^{LIN}(t)$ at frequency $n\omega_p$, V_{LO} and V_{dc} are the amplitudes of the Thevenin equivalent LO and dc voltage sources, and $Z_e(n\omega_p)$ is the impedance of the equivalent external embedding circuit at frequency $n\omega_p$, and $Z_c(n\omega_p) = -i/n\omega_p C_J$.

The iterative voltage update algorithm[12] proceeds as follows:

(i) Estimate the large signal voltage waveform across the nonlinear tunnel junction, V(t);
(ii) Using Eqs.(1) - (4), determine the current $I^{NL}(t)$ through the nonlinear element produced by the voltage V(t);
(iii) Kirchhoff's current law gives $I^{LIN}(t) = -I^{NL}(t)$;
(iv) Using a fast Fourier transform, obtain I_n^{LIN}, n=0, 1, 2,... from $I^{LIN}(t)$;
(v) Using Eqs. (5) - (7), obtain the V_n, n = 0, 1, 2,... consistent with these I_n^{LIN};
(vi) Using an inverse fast Fourier transform, obtain $V^{\#}(t)$ from this set of V_n;
(vii) Compare $V^{\#}(t)$ and V(t). If they are "equal", the iteration is complete. If not, set the new V(t) equal to $pV^{\#}(t) + (1-p)\ V(t)$ and return to step (ii). The convergence parameter, p, is normally a value in the range $0 < p \leqslant 1$ (Ref. 12).

This procedure is called the "voltage update method" because the mechanism for approaching the self-consistent solution in a controllable fashion is the updating of the junction voltage waveform with each iteration in step (vii). This method converges more rapidly the closer the linear embedding impedance is to a short circuit, and it may be shown[12] that it will always converge if the impedance of the linear embedding network is smaller in magnitude than that of the nonlinear circuit at all of the harmonic frequencies. This will be true for an SIS mixer whose $\omega_p R_N C_J$ product is larger than some moderate value, perhaps 0.5.

There is some discretion in choosing the convergence parameter p, to give a reasonable compromise between the speed of convergence and the possibility of divergence. In this work, the choice of p was fixed at unity since the use of an identity element proves sufficient to guarantee convergence. A resistive identity element consists of the parallel combination of a resistance R_{ID} and a resistance $-R_{ID}$. The net effect of the two parallel resistances on the circuit performance is zero since one cancels the effect of the other. However, in the large signal analysis, convergence is improved since the resistance R_{ID} is lumped in parallel with the linear embedding network and the resistance $-R_{ID}$ is added in parallel with the nonlinear circuit.[12] The effect of such an addition is to enhance the voltage update properties of the circuit by increasing the effective impedance of the tunnel junction and simultaneously decreasing the input impedance of the embedding network. From experience, setting R_{ID} equal to $0.5R_N$ has been found effective.

Small Signal Analysis

The small signal analysis is standard and will only be summarized here. In the linear approximation, the only frequencies which can produce a response at the IF frequency, ω_0, are the following sideband frequencies (we use here the notation of Saleh[16]):

$$\omega_n = \omega_0 + n\omega_p, \quad n = 0, \pm 1, \pm 2, \ldots, \pm N, \tag{8}$$

where the largest LO harmonic considered is $N\omega_p$. All lower sideband frequencies ($n < 0$) are seen to be negative, a convention which simplifies the equations. (For any physical linear circuit, $Z(-\omega) = Z^*(\omega)$).

The small-signal sideband currents and voltages at the pumped junction are related by a (2N+1) x (2N+1) admittance matrix whose elements Y_{mn} are given in Ref. 1, Eq. (7.5). The matrix $\| Y_{mn} \|$ is regarded as the admittance matrix of a multifrequency multiport network (see Ref. 17, Fig. 2) in which there is one port for every sideband frequency. Following Ref. 1, Eq. (7.2), the required quantities $W_{LO}(n\omega_p)$ are the Fourier components of U(t) in Eq. (3). The embedding impedance and junction capacitance are added in parallel with the intrinsic junction resulting in an "augmented network". The corresponding augmented admittance matrix is then inverted to give the impedance matrix:

$$\| Z_{mn} \| = \| Y_{mn} + [Z_e(\omega_n)^{-1} + i\omega_n C_J]\, \delta_{m,n} \|^{-1}. \tag{9}$$

The conversion efficiency from the signal port at frequency ω_1 to the IF at ω_0 is given by:

$$L^{-1} = 4R_e\, [Z_e(\omega_1)^{-1}]\, R_e\, [Z_e(\omega_0)^{-1}]\, |Z_{01}|^2 \tag{10}$$

and the (complex) output admittance at the IF is:

$$Y_{out} = Z_{00}{}^{-1} - Z_e(\omega_0)^{-1}. \tag{11}$$

Noise Analysis

There are three major noise sources known to contribute to the noise temperature of an SIS mixer: (i) shot noise due to the statistical nature of the current flow across the junction, (ii) thermal noise if the embedding network is at a finite temperature, and (iii) quantum noise due to the Heisenberg uncertainty principle. The problem of quantum noise in tunnel junction mixers, discussed in Ref. 9, has not been clearly resolved, and it has been ignored in the program. General considerations[18] require that a "high gain linear amplifier", such as the SIS mixer, add <u>at least</u> a half photon of fluctuation energy, referred to its input, to any incoming signal. This results in a <u>minimum</u> noise temperature $T_m = \hbar\omega/2k$, which is small at the frequencies of interest (2.8 K at 115 GHz).

The equivalent noise circuit of the SIS tunnel junction mixer contains a thermal noise source and a shot noise source at each sideband. The thermal noise is represented by a current generator at each frequency, whose magnitude is given by Ref. 1, Eq. (7.8). The thermal noise generators at the various sideband frequencies are uncorrelated, and so the noise powers can be directly converted to the IF and added to give the total ouput noise power due to thermal noise.

Shot noise is also characterized by a current generator at each frequency, whose amplitude is given in Ref. 1, Eq. (7.10). The various shot noise current generators are correlated[19,20] and so their effects at the IF must be added vectorially. The final contribution of the shot noise to the output noise of the mixer is given in Ref. 1, Eq. (7.13) and Eq. (7.16).

The total output noise power of the mixer is obtained by combining the thermal and shot noise components. The equivalent input noise temperature T_M of the mixer is then found by referring the total output power to the input of the mixer, and equating the power to kT_MB.

Typical Results

A FORTRAN computer program was developed to perform the large and small analysis described above. This program is described fully in Ref. 14, which will be available shortly. Several problems arise in adapting the nonlinear SIS circuit equations (Eqs. 1 - 4) for numerical solution. The minus infinity lower limit in Eq. 1 needs to be replaced by a suitably large negative number. The differential dt' needs to be replaced by a suitably small interval Δt. Finally, potential errors due to right-hand side terms in Eq. 1 being approximately equal yet opposite in sign must be evaluated. All three problems are discussed in Ref. 14, with suggested parameter values which gave accurate solutions in the cases we have investigated.

The program has been verified for the case of large junction capacitance by comparison with the three-frequency analysis. In a number of cases (not restricted to a large junction capacitance), the large signal waveforms predicted by this program were found to be consistent with those calculated by an independently developed SIS chaos program.[15] In addition, the waveforms determined from the voltage update method agreed with those from the multiple reflection technique. The CPU time on an AMDAHL V6 required for each full mixer analysis (large signal and small signal) is 0.5 min.

Although the program is still being tested and debugged, some preliminary results have been obtained which show the effect of finite junction capacitance on an SIS mixer. The experimentally measured I-V curve, shown in Fig. 2, of a two-junction Pb-alloy SIS element (fabricated by P. Timbie of Princeton University at NBS Boulder) for which $R_N = 72\ \Omega$ was assumed to be in a mixer circuit with a 113.9 GHz local oscillator. The intermediate frequency was 1.0 MHz and the IF load resistance was 50 Ω. In this low IF limit the signal, image, and LO source impedances must all be equal, and they were chosen to be $55 + i92\ \Omega$ (which includes C_J), the value which maximizes the conversion efficiency at a dc voltage in the center of the first photon step in the three-frequency approximation. This SIS mixer was analyzed for a wide range of junction capacitance. In each case the termination impedance at all higher harmonics and sidebands (not including the LO and its sidebands) was the parallel combination of the capacitive reactance and a resistance of 72 Ω (arbitrarily chosen equal to R_N).

For each value of capacitance the LO power and the dc voltage were optimized for maximum conversion on the first photon step. The largest value of mixer conversion gain, a few dB, was found in the limit of large capacitance, and agreed with the three-frequency approximation. The conversion gain decreased to a minimum of about unity at an $\omega_p R_N C$ product of unity, but then increased again for smaller $\omega_p R_N C$ values. The mixer noise temperatures corresponding to these results showed an even less dramatic variation with capacitance, remaining between 20 and 25 K. The mixer output impedance was negative at all capacitance values, while the input impedance was always positive. It must be emphasized that even if these conclusions are verified for this specific mixer circuit, they may be far from typical and should not be taken as general.

Conclusions

A FORTRAN computer program has been developed for analyzing SIS mixers with arbitrary embedding impedances at all LO harmonics and sidebands. This program has been verified using the three-frequency approximation and the multiple reflection algorithm. For the specific preliminary example considered in this paper, the optimum mixer performance is seen to occur for large or very small values of $\omega_p R_N C_J$. Low values of $\omega_p R_N C_J$ (roughly 0.3 to 2.0) lead to a deterioration in mixer performance.

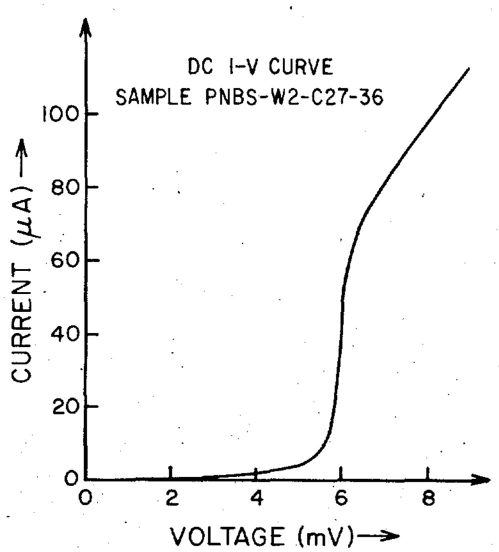

Figure 2. DC I-V Curve of SIS Device Used in the Example

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† NAS-NRC Research Fellow